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A study of displacement-based finite element approach for three-dimensional fluid-structure interaction in case of dissipative continua

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Abstract

Three basic problems of three-dimensional fluid-structure interaction dynamics are formulated: investigation of transient non-stationary processes, analysis of stationary harmonic oscillations, and finding of frequencies and shapes of natural vibrations. Based upon weak formulation of the mentioned problems and employing finite element method, a computational scheme and corresponding algorithms for solution of partial problem of finding frequencies and shapes of natural vibrations of three-dimensional coupled systems are constructed.

Previous computations using a displacement field and Lagrangian approximation to model the motion of liquid was confronted with some difficulties (Stucky and Lord [4]). In order to avoid them, we proposed to take natural viscosity of the medium into account.

A starting point for description of acoustic fluid motion is acoustic approximation of Navier-Stokes equations with preserving of the viscous stresses and linear viscoelasticity equations for the structure (Voytovych and Horlatch [5]).

An important class of problems is dealing with a very short-wave solution. In this case, introduction of viscosity to mathematical model is necessary for more realistic modeling due to the great influence of the viscosity of the medium on the propagation of high-frequency vibrations and importance of estimation of energy losses in practical applications. In case of eigenvalue problem, we can also evaluate the effect of viscosity.

1 Introduction

The urgency of study of acoustic processes in hydroelastic systems is connected with the large number of engineering applications in ship building and machine building, astronautics, medicine and other areas. A large amount of work has been devoted to this subject during the last years. Let us mention for instance (Morand and Ohayon [2]), where numerical methods and further references are given. In the present paper we are interested in the situation where damping arises in the propagation media itself, due to friction. A general approach to this topic can be found in the book by Landau and Lifshitz [1].

In fluid-structure interaction problems, since the solid is generally described in terms of displacements in order to provide the same variable for the fluid, this approach presents advantages because compatibility and equilibrium through the fluid-solid interface are satisfied automatically. This approach leads to sparse symmetric matrices and cold be applied to the solution of a broad range of problems.

Our main instruments for numerical analysis of time-domain dissipative acoustic problem are Galerkin semidiscretization of appropriate evolutionary variational problems in space, one-step time integration algorithms (Shynkarenko [3]) and energy analysis of stability and convergence for these procedures.

In this paper we also considered quadratic eigenvalue problem arising during determination of the vibration modes of an acoustic fluid that interacted with elastic solid in case of viscous medium. A numerical discretization based on Lagrangian finite elements to approximate viscous vibration modes of coupled systems is analyzed. The advantage of this method is that it does not require any previous manipulation to be solved by standard eigensolvers.

In addition to this, we analyzed the acoustic fluid-structure interaction problem in case of stationary harmonic oscillations for viscous medium.

2 Statement of the problem

Let us consider as a problem, a viscous compressible fluid contained into a linear isotropic elastic (three-dimensional) structure which obeys Hooke's law. The domains occupied by the fluid and the solid are denoted by Ω^F and Ω^S, respectively. Let $\Gamma^C = \Gamma^S \cap \Gamma^F \neq \emptyset$ be the interface between both media and ν its unit normal vector pointing outwards Ω^S (Γ^S and Γ^F are the boundaries of solid and fluid, respectively). The exterior boundary of the solid is the union of two parts, Γ^{SU} and Γ^{SP}: the structure is fixed on Γ^{SU} and free on Γ^{SP}.

We use the following notation for physical magnitudes in the fluid:

- u^F: displacement vector,
- ρ^F: density,
- c: sound velocity,
- η^F, ξ^F: first and second coefficient of dynamic viscosity,
- $\{a^F_{ijkm}(x)\}^3_{ijkm=1}$ and $\{c^F_{ijkm}(x)\}^3_{ijkm=1}$: modal coefficient of elasticity and viscosity defined by

$$a^F_{ijkm} = \begin{cases} \rho^F c^2, & \text{for } i=j=k=m, \\ 0, & \text{in other cases,} \end{cases} \quad c^F_{ijkm} = \begin{cases} 2\eta^F, & \text{for } i=j=k=m, \\ \xi^F + 4\eta^F/3, & \text{for } i=j,\, k=i,\, m=j, \\ \xi^F - 2\eta^F/3, & \text{for } i=j,\, k=m,\, i\neq k, \\ 0, & \text{in other cases,} \end{cases}$$

- $\sigma^F(u^F)$: stress tensor defined by $\sigma^F_{ij}(u^F) = a^F_{ijkm}\, e_{km}(u^F) + c^F_{ijkm}\, e_{km}((u^F)')$

and in the solid:

- u^S: displacement vector,
- ρ^S: density,
- λ and μ: Lame coefficients,
- η^S, ξ^S: first and second coefficient of dynamic viscosity,
- $e(u^S)$: strain tensor defined by $e_{ij}(u^S) = (u^S_{i,j} + u^S_{j,i})/2$
- $\{a^F_{ijkm}(x)\}^3_{ijkm=1}$ and $\{c^F_{ijkm}(x)\}^3_{ijkm=1}$: modal coefficient of elasticity and viscosity defined by

$$a^S_{ijkm} = \begin{cases} 2\mu, & \text{for } i=k,\, j=m,\, i\neq j, \\ \lambda+2\mu, & \text{for } i=j,\, k=i,\, m=j, \\ \lambda, & \text{for } i=j,\, k=m,\, i\neq k, \\ 0, & \text{in other cases,} \end{cases} \quad c^S_{ijkm} = \begin{cases} 2\eta^S, & \text{for } i=k,\, j=m,\, i\neq j, \\ \xi^S+2\eta^S, & \text{for } i=j,\, k=i,\, m=j, \\ \xi^S, & \text{for } i=j,\, k=m,\, i\neq k, \\ 0, & \text{in other cases,} \end{cases}$$

- $\sigma^S(u^S)$: stress tensor, which we assume related to the strains by Hooke's law, i.e.

$$\sigma^S_{ij}(u^S) = a^S_{ijkm}\, e_{km}(u^S) + c^S_{ijkm}\, e_{km}((u^S)'),$$

In all cases repeated indices denote summations from 1 to 3 and $(\circ)' = \frac{\partial}{\partial t}(\circ)$, $(\circ)_{,i} = \frac{\partial}{\partial x_i}(\circ)$.

Let $\Omega = \Omega^S \cup \Omega^F$ and

$$u = \begin{cases} u^S(x), & x \in \Omega^S \\ u^F(x), & x \in \Omega^F \end{cases}$$

The governing equation for small amplitude motions of the coupled system are the following:

$$\rho u_i'' - \sigma_{ij,j} = f_i \quad \text{in} \quad \Omega \times (0, T] \tag{1}$$

with the following boundary conditions

$$\begin{aligned} u_i = 0 \qquad & \text{on} \qquad \Gamma^{SU} \times [0, T], \quad \text{mes}(\Gamma^{SU}) > 0, \\ \sigma_{ij}\nu_j = \hat{\sigma}_i \qquad & \text{on} \qquad \Gamma^{SP} \times [0, T], \end{aligned} \tag{2}$$

plus initial conditions

$$u|_{t=0} = u^0, \quad u'|_{t=0} = u^1$$

3 Three problems of dissipative acoustics

3.1 Time-domain problem

To obtain a variational formulation of this problem, we consider the following spaces:

$$V = \{v \in H^1(\Omega)^3 | v = 0 \text{ on } \Gamma^{SU}\}, \quad H = L^2(\Omega)^3$$

If we multiply the equation in (1) by test function from V, by using Green's formulas and taking into account boundary conditions (2), we are led to the following weak form of time-domain problem.

$$\begin{cases} \text{given} \qquad l \in L^2(0, T; V'),\ u^0 \in V,\ u^1 \in H \\ \text{find} \qquad u \in L^2(0, T; V) \quad \text{such that} \\ m(u''(t), v) + c(u'(t), v) + a(u(t), v) = < l(t), v > \qquad \forall\, t \in (0, T],\ \forall\, v \in V \\ (u(0) - u^0, v)_{H^1(\Omega)} = 0 \\ m(u'(0) - u^1, v) = 0 \end{cases} \tag{3}$$

The matrices associated to the bilinear forms and linear functional are defined by:

$$\begin{cases} m(u, v) &= \int\limits_\Omega \rho u_i v_i \,\mathrm{d}x, \\ a(u, v) &= \int\limits_\Omega a_{ijkm} e_{ij}(u) e_{ij}(v) \,\mathrm{d}x, \\ c(u, v) &= \int\limits_\Omega c_{ijkm} e_{ij}(u) e_{ij}(v) \,\mathrm{d}x, \\ < l, v > &= \int\limits_\Omega f_i v_i \,\mathrm{d}x + \int\limits_{\Gamma^{SP}} \hat{\sigma}_i v_i \,\mathrm{d}\gamma. \end{cases} \tag{4}$$

For the space discretization we use classical Lagrangian finite elements for both media. For the time discretization of the differential system (3), we will apply one-step time integration algorithm of order 2 with respect to the time step (Shynkarenko [3]).

3.2 Harmonic oscillation

We considered the acoustic fluid-structure interaction problem in the case of stationary harmonic oscillation for viscous media. Our main instrument for numerical analysis is Galerkin discretization of appropriate variational problem in space. As a result, we obtain the following simultaneous equations:

$$\begin{bmatrix} A - \omega^2 M & -\omega C \\ \omega C^T & -\omega^2 M + A \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}, \tag{5}$$

where ω is the forced vibration angular frequency, U_1 and U_2 are the real and imaginary part of the solution, f_1 and f_2 are the real and imaginary part of distributed source of sound. Quantities A, C and M are the stiffness, damping and inertia matrices of the finite element assemblage, respectively. Since this matrices are symmetric and positively defined, we can uniquely solve this system (Voytovych [6]).

3.3 Natural vibrations

When the problem of vibration of an elastic solid coupled with viscous compressible acoustical fluid is modeled by means of finite element method, the natural frequencies and mode shapes are the solution of the following quadratic eigenvalue problem:

$$(A + \lambda C + \lambda^2 M)u = 0,$$

where λ and u represent the associated eigenpairs. The eigenproblem may be rewritten in the following linearized form:

$$(B - \lambda D)p = 0$$

with

$$p = \{u, \lambda u\}^T, \quad B = \begin{bmatrix} 0 & A \\ A & C \end{bmatrix}, \quad D = \begin{bmatrix} A & 0 \\ 0 & -M \end{bmatrix}.$$

We use the Matlab eigensolver *eigs* (based on Arnoldi iteration) for solving the involved spectral problems.

4 Concluding remarks

Within suggested methodology with application of common FEM approximations the software implementation for 3D case was developed. A number of model problems that prove effectiveness of proposed technique were considered. Natural frequencies of spatial shell structures that interacted with viscous fluids or without interaction are compared to results of other authors obtained in terms of mathematical models of Kirchgoff-Love and Timoshenko shell theories and two-dimensional formulations of elasticity theory. Also we can observe the usefulness to have the viscosity in the equation for improving of the approximation features of numerical schemes (Voytovych at al. [7]).

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